Sizing Linear and PWM Amplifiers Driving a Brush-Type Motor

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***Abstract*—This application note provides a design process for sizing single-phase linear and PWM amplifiers driving a brush-type motor (BTM). Design inputs consist of the motor torque constant, back-emf constant, resistance and inductance, load inertia, and worst-case angular velocity and load torque profiles. The design outputs are the key amplifier requirements: amplifier bus voltages, peak output current, continuous output current, peak output power, and continuous power dissipation. The *peak output power* and *continuous power dissipation* calculations apply only to linear amplifiers as the current specifications in PWM amplifiers are sufficient to determine power requirements. Special attention is paid to trapezoidal angular velocity profiles and piecewise-constant load force profiles, which are used in a companion spreadsheet [1]. Equations for power supply sizing and motor heating power are also provided.**

***Index Terms*—Amplifier Sizing, Brush-Type Motor, Varedan Document 4083-42-008 Revision C**

I. Introduction

This document provides background and design equations for sizing linear and PWM amplifiers driving a linear brush-type motor (BTM). The companion design spreadsheet [1] incorporates the design equations of Section IV below and provides an easy-to-use design tool that will meet the needs of most designers. However, the analyses contained here can provide valuable context and help the designer understand the limits of the design spreadsheet.

It is useful to think of amplifier sizing as a design process where there are design inputs and design outputs. For a BTM system design, the following design inputs are used to size a linear amplifier. SI units are used throughout unless otherwise noted.

- Motor torque constant K_τ $(N-m/A_{rms})$
- Motor back-emf constant K_e $(V_{peak}/(rad/sec))$
- Motor resistance R (Ω)
- Motor inductance L (H)
- Motor load inertia J $(kg-m^2)$
- Worst-case angular velocity profile $\omega(t)$ (rad/sec)
- Worst-case load torque profile $\tau_{load}(t)$ $(N-m)$

The design methodology described herein produces design outputs that consist of the key amplifier requirements below. *Peak output power* and *continuous power dissipation* are design outputs that apply only to linear amplifier sizing and are marked with an asterisk (*). Specifications for the output current in PWM amplifiers are sufficient to determine power requirements. The amplifier requirements are:

- Bipolar Amplifier Bus Voltage $\pm B$ for linear amplifiers, (Amplifier Bus Voltage $2B$ for PWM amplifiers)
- Peak Output Current I_{peak}
- Continuous Output Current I_{cont}
- Peak Output Power* P_{peak}
- Continuous Power Dissipation* P_{cont}

Once the amplifier requirements are determined, an amplifier is sized with specifications that meet or exceed the requirements.

The remainder of this application note is organized as follows: In Section II, the design inputs and outputs are discussed and clearly defined. Algebraic equations that relate the design inputs to the design outputs are presented in Section III, and these equations are specialized to a trapezoidal angular velocity profile and a piecewise-constant load torque profile in Section IV. The equations of Section IV are implemented in the spreadsheet [1]. Section V contains some simple design checks. Section VI works through a sizing example and Section VII addresses power supply sizing and motor ohmic heating. Concluding remarks are found in Section VIII.

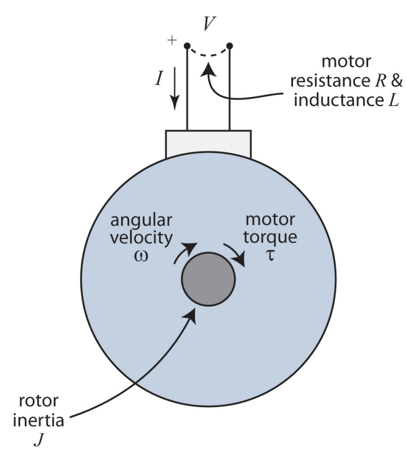

Fig. 1. Schematic diagram of a brush-type motor (BTM).

II. Background & Design Inputs

The notation describing a brush-type motor is introduced in this section together with definitions of the design inputs used for linear and PWM amplifier sizing. Consider first the

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simplified schematic of a BTM depicted in Figure 1, which helps visualize the motor parameters.

The current in the motor windings produces a torque, and the motion of the motor rotor induces a back-emf voltage on the terminals. These behaviors are quantified by the motor torque constant and the motor back-emf constants. These and other inputs are defined as follows.

A. Motor torque constant K_τ $(N-m/A_{rms})$

The motor torque constant K_τ relates the motor current I to the motor torque F according to

$$F = K_\tau I. \tag{1}$$

The motor torque is the sum of the inertial torque and load torque. More precisely, by defining the angular acceleration by $\alpha(t) \equiv d\omega/dt$ one gets

$$F = J\alpha(t) + \tau_{load}. \tag{2}$$

B. Motor back-emf constant K_e $(V/(rad/s))$

The motor creates an electromotive force (emf) which is considered backward by common sign conventions. If the generator terminals are disconnected or otherwise unloaded, the terminal voltage is related to the back-emf constant K_e and the motor angular velocity ω by

$$V = K_e \omega. \tag{3}$$

C. Coil resistance R and inductance L

Figure 1 shows the motor as having resistance R and inductance L which are measured across the motor terminals. The inductance is neglected in most of this application note except in the checks where the validity of this approximation is assessed.

D. Inertia J $(Kg-m^2)$

The inertia J is the total moving inertia internal and external to the BTM. The internal inertia is usually listed on the motor datasheet.

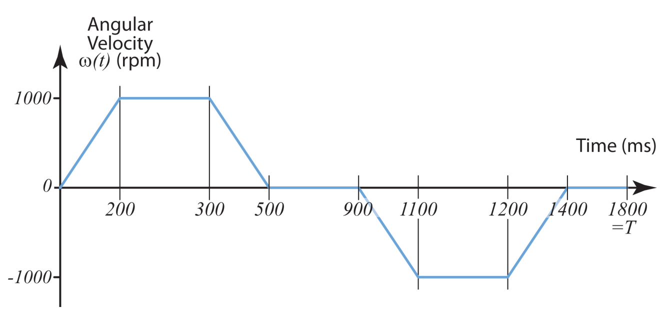

Fig. 2. Typical worst-case trapezoidal angular velocity profile – a design input.

E. Worst-case angular velocity and load torque profiles

A necessary design input for selecting an amplifier is one or more worst-case angular velocity and load torque profiles. The accelerations, load torques, peak velocities, and other features of these profiles inform the amplifier selection. In choosing worst-case profiles it is useful to understand how they affect the various design outputs. Further, such understanding can often guide the system design in that the design engineer may alter motion and load trajectories to reduce amplifier cost. A qualitative discussion of how angular velocity and load torque profiles impact each of the design outputs is offered in subsections 1-7 below.

1) Peak Output Current: Short-time-constant thermal limits on amplifier interconnects constrain the amplifier peak output current. Since large currents are needed to produce high torques, angular velocity profiles with high peak accelerations will have demanding requirements for peak output current. Even a generally low-angular-velocity low-acceleration profile can be challenging in terms of peak output current if there are short bursts of high acceleration.

2) Continuous Output Current: Heating and long thermal time constants associated with amplifier conductors and other components can limit amplifier performance. Motions that repeatedly accelerate and decelerate the load inertia, or apply large load torques, can cause overheating by exceeding the continuous output current specification of an amplifier.

3) Peak Output Power:* Used in sizing linear amplifiers only, the peak output power is that peak power experienced by a single output transistor. Heavy braking or other large motor torques at high speeds cause large voltage drops and high currents in the output transistors, resulting in high power dissipation.

4) Continuous Power Dissipation:* Used in sizing linear amplifiers only, the continuous power dissipation is the time-averaged power collectively dissipated by all the output transistors. Trajectories that repeatedly impose high inertial or load torques cause high continuous power dissipation in the output transistors. Over time, the temperature of the heat sink rises and the junction temperature of the power transistors can exceed specifications. Such trajectories also challenge the continuous output current specification.

5) Bipolar Amplifier Bus Voltage for linear amplifiers, Amplifier Bus Voltage for PWM amplifiers: The bus-to-bus voltage in a linear or PWM amplifier must exceed the largest back-emf voltages. Thus, high-speed trajectories and/or motors with large back-emf constants can exceed the capabilities of the power supply/amplifier system. In linear amplifiers, the bus voltages are $+B\,Volts$ and $-B\,Volts$ and are referred to as the "bipolar amplifier bus voltage $\pm B$". In PWM amplifiers, the bus voltages are $0\,Volts$ and $2B\,Volts$ where the latter is normally referred to as the bus voltage and the bus at $0\,Volts$ is implied. The variable B is

*Calculations marked with an asterisk (*) only apply to linear amplifiers used in specifying both amplifier types in order to simplify notation. In either case, the bus-to-bus voltage is $2B$.

6) Trapezoidal Velocity Profiles: A common trapezoidal angular velocity profile will be used to estimate the key amplifier requirements. Trapezoidal profiles can be constructed to produce any of the demands described in subsections 1-5 above and approximate many of the motions that the designer may wish to generate. Figure 2 depicts an example of a trapezoidal profile with the timing and amplitude variables that precisely describe the motion. The analysis in Section IV refers to the profile of Figure 2.

The trapezoidal profile is assumed to be periodic with period T which is 1.80 seconds for the profile in Figure 2.

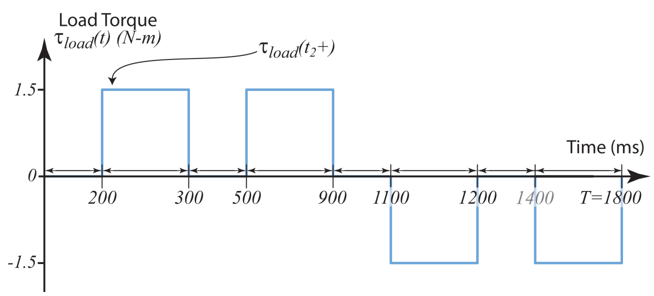

Fig. 3. Example of a worst-case piecewise-constant load torque profile – a design input. $\tau_{load}(t_k+)$ denotes the torque immediately following time t_k.

7) Piecewise-Constant Load Torque Profiles: In addition to inertial torques required to accelerate the inertia J, the motor may also be used to apply, for example, cutting torques in a machining operation. For the sake of amplifier sizing, the load-torque profile $\tau_{load}(t)$ is assumed to be piecewise constant and have transition times at the corner times of the angular velocity profile. See Figure 3 for an example of a piecewise-constant torque profile. In referring to the constant torque levels in the profile, we define $\tau_{load}(t+)$ to be the torque immediately following time t. For example, in the profile in Figure 3, $\tau_{load}(50ms+) = 1.5N$, and $\tau_{load}(600ms+) = 0N$.

III. Design Outputs

In this section, the design outputs are expressed, via equations, in terms of the design inputs. These equations serve as design tools for sizing the linear amplifier.

A. Bipolar Amplifier Bus Voltage $\pm B$ for linear amplifiers, Amplifier Bus Voltage $2B$ for PWM amplifiers

The bipolar amplifier bus voltage $\pm B$ in linear amplifiers determines the limits of the output voltage range. The positive bus voltage is $+B$ or simply B, the negative bus voltage is $-B$, and the voltage between the buses is $2B$. For PWM amplifiers the bus voltages are 0 and $2B$ so that the voltage between the buses is also $2B$. For both types of amplifiers, the bus voltages limit the voltage across the BTM terminals to $\pm 2B$. Some Varedan linear amplifiers have half-bridge outputs where one motor terminal is connected to the power supply neutral. Half-bridge amplifier model numbers end with an I and the calculations herein must be modified accordingly.

Delaying discussion of the coil inductance until later, the absolute value of the peak output voltage on the BTM terminals is given by

$$V_{peak} = \max_t |K_e\omega(t) + \frac{R}{K_\tau}(\tau_{load}(t) + J\alpha(t))|, \quad (4)$$

where $\max_t x(t)$ is the maximum value of $x(t)$ achieved for relevant values of t. A margin of safety of, say, 20% is often included to account for variations in line voltage and other uncertainties. Thus, the bipolar amplifier bus voltage might be chosen to be

$$\pm B = \pm 1.2V_{peak}. \quad (5)$$

The corresponding bus voltage for a PWM amplifier is

$$V_{bus} = 2B = 2.4V_{peak}. \quad (6)$$

B. Peak Output Current I_{peak}

The peak output current is the maximum absolute value of the current attained over the motion defined by the angular velocity and load torque profiles. The peak output current is given by

$$I_{peak} = \max_t |(J\alpha(t) + \tau_{load}(t))|/K_\tau. \quad (7)$$

C. Continuous Output Current I_{cont}

Long-time-constant thermal limits affect the maximum continuous output current that an amplifier can supply. This specification is expressed as an rms value. Continuous operation above this threshold is expected to overheat conductors or other components. For non-constant operation, the specification is interpreted as an rms current computed over an extended period of time. Such an interpretation is taken to be valid for periodic trajectories whose period is less than the thermal time constant of the heat sink ($\sim$ 60 seconds).

The continuous output current requirement for a periodic acceleration profile $\alpha(t)$ and load-torque profile $\tau_{load}(t)$, both having period T, is given by

$$I_{cont} = \left(\frac{1}{T}\int_0^T \left(\frac{J\alpha(t) + \tau_{load}(t)}{K_\tau}\right)^2 dt\right)^{1/2}. \quad (8)$$

D. Peak Output Power* P_{peak}

For single-phase linear amplifiers, the BTM terminals are differentially driven up to a voltage of $\pm 2B$. Two transistors (or transistor sets when devices are paralleled) are conducting at the same time and each transistor experiences an equal voltage drop. There is a virtual neutral at the midpoint of the coil and each side of the H-bridge sees half the voltage on the coil. Neglecting the coil inductance, the output power dissipated in one of conducting transistors in a linear amplifier is

$$B|I(t)| - K_e\omega(t)I(t)/2 - I(t)^2R/2. \tag{9}$$

Using the relationship $I(t) = (J\alpha(t) + \tau_{load}(t))/K_\tau$ and taking the maximum yields

$$P_{peak} = \max_t \left[\frac{B}{K_\tau}|J\alpha(t) + \tau_{load}(t)| - \frac{K_e}{2K_\tau}\omega(t)\times (J\alpha(t) + \tau_{load}(t)) - \left(\frac{J\alpha(t) + \tau_{load}(t)}{K_\tau}\right)^2 \frac{R}{2}\right] \tag{10}$$

The amplifier output transistors have a limit on instantaneous power dissipation, and this limit is approached when there is a large voltage drop across an output transistor while a high current is passing through that transistor. Such a high-voltage high-current condition arises when the motor angular velocity is large in magnitude, either positive or negative, and when the motor aggressively brakes or when large load torques are applied. In this situation, the large back-emf and the bus voltage add constructively across the output transistor, and large currents are required. However, the voltage across the output transistor is reduced by the voltage drop across the coil resistance R. Inductance is neglected in the calculation above although it can be an issue in high-inductance motors.

This peak output power calculation does not apply to PWM amplifier sizing.

E. Continuous Power Dissipation* P_{cont}

The continuous power dissipation in linear amplifiers has the potential to overheat the heat sink system and hence the transistor junctions. The continuous power dissipation for a periodic angular velocity profile $\omega(t)$ is taken to be the average over the period T of the power dissipated in the output transistors. Integrating the power equation inside the max above and multiplying by 2 to capture the power in the 2 transistors conducting at any point in time yields

$$P_{cont} = \frac{1}{T}\int_0^T \left(\frac{2B}{K_\tau}|J\alpha(t) + \tau_{load}(t)| - \frac{K_e\omega(t)}{K_\tau}\times (J\alpha(t) + \tau_{load}(t)) - R\left(\frac{J\alpha(t) + \tau_{load}(t)}{K_\tau}\right)^2\right) dt. \tag{11}$$

This continuous power dissipation calculation does not apply to PWM amplifier sizing.

IV. Evaluating the Design Outputs for Trapezoidal Velocity Profiles

Note that a trapezoidal angular velocity profile is equivalent to a piecewise-constant acceleration and inertial torque profile. By focusing on trapezoidal angular velocity profiles and piecewise-constant load torque profiles, the design equations simplify and a systematic design process can be implemented in the companion spreadsheet [1]. Refer to the periodic trapezoidal angular velocity profile in Figure 2 and note that the profile can be specified by the nine corners in the trapezoidal profile that have (time, angular velocity) coordinates (0s,0rpm), (0.2s, 1000rpm), (0.15s, 1000rpm),. . . , (1.8s,0rpm), or, for short, $(t_k, \omega(t_k))$; $k = 1, 2, \ldots, 9$. Similarly for $(t_k, \tau_{load}(t_k+))$; $k = 1, 2, \ldots, 9$, where $\tau_{load}(t_k+)$ denotes the load torque just after the time t_k. Due to discontinuities at t_k, the value of the load torque at t_k is not well defined. Again, the trajectory is assumed to be periodic with, in the case of Figures 2 and 3, a period of $T = 1.8s$.

The first observation to make is that the design outputs can be calculated using $\omega(t)$ and $\tau_{load}(t)$ near the corner points of the angular velocity profile. Specifically, the design outputs are determined by the velocities $\omega(t_k)$, and the acceleration $\alpha(t)$ just before and just after each corner, which are denoted $\alpha(t_k-)$ and $\alpha(t_k+)$ respectively. The torques $\tau_{load}(t_k+)$ and $\tau_{load}(t_k-)$ also enter the calculations where the inertial torque due to acceleration and load torques are combined into a single torque. Since the trajectory is periodic, the acceleration just after the 9th corner is the same as that just after the 1st corner. That is $\alpha(t_9+) = \alpha(t_1+)$. The acceleration and the load torques are often discontinuous right at the corners and not defined there – although inductance in the motor windings and other sources of filtering will round the corners in practice.

When restricted to trapezoidal angular velocity profiles, the five design equations above are simplified as follows.

A. Bus Voltages for a Trapezoidal Profile

The peak voltage on the BTM terminals amongst all the profile corners is obtained by maximizing over all values of the corner velocities $\omega(t)$ and the combined inertial and load torques $(J\alpha(t_k-)+\tau_{load}(t_k-))$ and $(J\alpha(t_k+)+\tau_{load}(t_k+))$ just before and just after the corners. Thus, the calculation involves $8 \times 2 = 16$ different evaluations of the absolute value expression below where each sign is evaluated at each corner.

$$V_{peak} = \max_k \max_\pm |K_e\omega(t_k) + \frac{R}{K_\tau}(J\alpha(t_k\pm) + \tau_{load}(t_k\pm))|. \tag{12}$$

In the maximization with respect to the sign $\pm$, either the positive or negative sign is used throughout. Applying a recommended margin of 20% yields

$$\pm B = \pm 1.2 V_{peak} \tag{13}$$

for the bipolar amplifier bus voltage for a linear amplifier. The bus voltage for a PWM amplifier is

$$V_{bus} = 2B = 2.4 V_{peak}. \tag{14}$$

B. Continuous Output Current for a Trapezoidal Profile

The acceleration and load torque are constant between corner times in a trapezoidal angular velocity profile. Thus, the integral expression for the continuous output current becomes the summation

$$I_{cont} = \left(\frac{1}{T} \sum_{k=1}^{8} \left(\frac{J\alpha(t_k+) + \tau_{load}(t)}{K_\tau} \right)^2 (t_{k+1} - t_k) \right)^{1/2}, \tag{15}$$

where $T = t_9$ is the period of the angular velocity profile. Observe that $\alpha(t_k+)$ is the value of $\alpha(t)$ in the time interval between t_k and t_{k+1} and similarly for $\tau_{load}(t_k+)$.

C. Peak Output Power* for a Trapezoidal Velocity Profile

For linear amplifiers, peak output power is typically required just after a corner point in the trapezoidal profile where there is high speed and heavy braking. However, because of the piecewise-constant load torques under consideration, it is possible that the peak is attained just prior to a corner time. Thus, the maximization of the peak power expression, for a single transistor, is over all points in time that are just before and just after the corner times:

$$\begin{aligned} P_{peak} = \max_k \max_\pm \Bigg[& \frac{B}{K_\tau} |J\alpha(t_k\pm) + \tau_{load}(t_k\pm)| \\ & - \frac{K_e}{2K_\tau} \omega(t_k) \left(J\alpha(t_k\pm) + \tau_{load}(t_k\pm) \right) \\ & - \left(\frac{J\alpha(t_k\pm) + \tau_{load}(t_k\pm)}{K_\tau} \right)^2 \frac{R}{2} \Bigg]. \end{aligned} \tag{16}$$

This peak output power calculation does not apply to PWM amplifier sizing.

D. Continuous Power Dissipation* for a Trapezoidal Velocity Profile

Integrating the expression for continuous power dissipation for a trapezoidal angular velocity profile yields

$$\begin{aligned} P_{cont} = \frac{1}{T} \sum_{k=1}^{8} \Bigg(& \frac{2B}{K_\tau} |J\alpha(t_k+) + \tau_{load}(t_k+)| \\ & - \frac{K_e}{K_\tau} \left(\frac{\omega(t_k) + \omega(t_{k+1})}{2} \right) (J\alpha(t_k+) + \tau_{load}(t_k+)) \\ & - \left(\frac{J\alpha(t_k+) + \tau_{load}(t_k+)}{K_\tau} \right)^2 R \Bigg) (t_{k+1} - t_k). \end{aligned} \tag{17}$$

This continuous power dissipation calculation does not apply to PWM amplifier sizing.

V. Design Checks

In implementing the design equations above, there are various approximations to be justified and ways that errors may occur. First, one must check that the BTM inductance is sufficiently small and that the electrical time constant is small relative to the time intervals in the motion and torque profiles. Further, confusion may arise from the unit systems used. Thus, it is worth performing the few simple checks outlined below.

A. Inductance Check

The electrical time-constants of brush-type motors are often short relative to transition times for coil currents. Thus, the inductance is not included in the calculation of bus voltages above. However, a simple check can be performed to insure that the additional voltage LdI/dt due to the coil inductance can be supplied by the bus voltages. The simple angular acceleration and load torque profiles used in sizing would require an impulsive current profile for perfect tracking. Such infinite impulses are not required in practice, however, as S-curve velocity profiles may be used or the natural corner rounding due to the inductance may be desirable. A reasonable goal is for the current to settle, following a corner time t_k, within roughly 15% of the subsequent interval duration $t_{k+1} - t_k$. The voltage on the motor terminals is given by

$$V = RI + L\frac{dI}{dt} + K_e\omega. \tag{18}$$

Define the values for I just before and just after t_k by

$$I(t_k-) = (m\alpha(t_k-) + F_{load}(t_k-))/K_f, \tag{19}$$

and

$$I(t_k+) = (m\alpha(t_k+) + F_{load}(t_k+))/K_f. \tag{20}$$

The approximate value $\tilde{I}(t_k)$ for the current at t_k is defined to be the average of these two quantities:

$$\tilde{I}(t_k) = \frac{1}{2}(I(t_k-) + I(t_k+)). \tag{21}$$

Using the 15% settling-time approximation leads to an approximation $\frac{\tilde{d}}{dt}I$ for the rate of change of the current at t_k:

$$\frac{\tilde{d}}{dt}I(t_k) \approx (I(t_k+) - I(t_k-))/(0.15(t_{k+1} - t_k)). \quad (22)$$

Since the H-bridge output can apply any voltage in the range $-2B$ to $+2B$, one half of the estimated requisite coil voltage should lie in the interval $[-B, +B]$. That is, one should check that

$$\frac{1}{2}V \approx \frac{1}{2}\left[R\tilde{I}(t_k) + L\frac{\tilde{d}}{dt}I(t_k) + K_e v(t_k)\right] \in [-B, +B]. \quad (23)$$

B. Verify that $K_e = K_\tau$

The back-emf constants and torque constants in permanent-magnet motors are proportional to each other. In the case of BTMs, the constant of proportionality is 1 provided SI or other consistent units are used. Some minor error in this relationship might occur due to BTM nonlinearity or measurement error.

C. Verify that $\frac{L}{R} = \tau_e$

Motor datasheets are redundant when it comes to quoting resistance, inductance, and motor electrical time constant τ_e. The relationship above holds up to round-off and measurement error. If millihenries are used instead of henries and milliseconds are used instead of seconds in the calculations, errors will escape detection with this check.

VI. Sizing Example

To illustrate the use of the design equations, consider the amplifier sizing to address the following design inputs:

- Motor torque constant $K_\tau = 0.362\, N - m/A_{rms}$
- Motor back-emf constant $K_e = 0.362\, V/(rad/s))$
- Motor resistance $R = 1.0\,\Omega$
- Motor inductance $L = 9\, mH$
- Motor load inertia $J = 0.0088\, kg - m^2$
- Worst-case angular velocity profile $\omega(t)$ given by Figure 2.
- Worst-case load torque profile $\tau_{load}(t)$ given by Figure 3.

A. Example: Bus Voltages

The peak output voltage occurs when acceleration and angular velocity are both high. This peak occurs in the profile of Figure 2 just prior to corner 2 and just prior to corner 6 when the peak voltage reaches the same value. The acceleration prior to corner 2 is $(1000\, rpm - 0\, rpm)(2\pi\, rad/rev)(1\, min/\, 60sec)/(0.2\, sec) = 523.6\, rad/sec$. Thus, using the formula for V_{peak}, one obtains:

$$\begin{aligned} V_{peak} &= |K_e\omega(t_2) + \frac{R}{K_\tau}\left(J\alpha(t_2-) + \tau_{load}(t_2-)\right)| \\ &= |0.362 \cdot 104.7 + \frac{1.0}{0.362} \cdot (0.0088 \cdot 523.6 + 0)| \\ &= 50.6\, V \end{aligned} \quad (24)$$

Applying a margin of 20% and dividing by two yields, for a linear amplifier,

$$\pm B = \pm 1.2 \cdot 50.6/2 = \pm 30.4\, V. \quad (25)$$

For a PWM amplifier, one has

$$2B = 60.7\, V. \quad (26)$$

B. Example: Peak Output Current

The peak current occurs at the peak of the motor torque (inertial + load). Since the inertial torque is large in this example, the peak output current achieves the same large value following corners 1,3,5, and 7. The formula for a trapezoidal profile yields

$$\begin{aligned} I_{peak} &= \frac{|J\alpha(t_1+) + \tau_{load}(t_1+)|}{K_\tau} \\ &= \frac{0.0088 \cdot 523.6 + 0}{0.362} = 12.73\, A. \end{aligned} \quad (27)$$

C. Example: Continuous Output Current

The current during the acceleration ramps is the peak value of $12.73\, A$ as computed above. The current during the hold periods (e.g. the current at t_2+) is

$$I = \frac{|J\alpha(t_2+) + \tau_{load}(t_2+)|}{K_\tau} = \frac{0.0088 \cdot 0 + 1.5}{0.362} = 1.28\, A. \quad (28)$$

The continuous current is therefore

$$\begin{aligned} I_{cont} &= \\ &\left(\frac{1}{T}\sum_{k=1}^{8}\left(\frac{\alpha(t_k+) + \tau_{load}(t_k+)}{K_\tau}\right)^2 (t_{k+1} - t_k))\right)^{\frac{1}{2}} \\ &= \left(\frac{1}{1.8}(12.73^2 \cdot (4 \cdot 0.05) + 1.28^2 \cdot (2 \cdot 0.1 + 2 \cdot 0.4))\right)^{\frac{1}{2}} \\ &= 9.03\, A_{rms}. \end{aligned} \quad (29)$$

D. Example: Peak Output Power*

For linear amplifiers, he peak output power occurs just after corners 3 and 7 where the power reaches the same high level. The peak power formula becomes

$$\begin{aligned}
P_{peak} =& \frac{B}{K_\tau}|J\alpha(t_3+) + \tau_{load}(t_3+)| \\
&- \frac{K_e}{2K_\tau}\omega(t_3)(J\alpha(t_3+) + \tau_{load}(t_3+)) \\
&- \left(\frac{J\alpha(t_3+) + \tau_{load}(t_3+)}{K_\tau}\right)^2 \frac{R}{2} \\
=& \frac{30.4}{0.362}|0.0088 \cdot (-523.6) + 0| \\
&- \frac{0.362}{2 \cdot 0.362} \cdot 104.7 \cdot (0.0088 \cdot (-523.6) + 0) \\
&- \left(\frac{0.0088 \cdot (-523.6) + 0}{0.362}\right)^2 \cdot \frac{1.0}{2} \\
=& 547\, W.
\end{aligned} \tag{30}$$

This calculation does not apply to PWM amplifiers.

E. Example: Continuous Power Dissipation*

For linear amplifiers, the continuous power dissipation integral becomes for a trapezoidal angular velocity profile:

$$\begin{aligned}
P_{cont} =& \frac{1}{T}\sum_{k=1}^{8}\left(\frac{2B}{K_\tau}|J\alpha(t_k+) + \tau_{load}(t_k+)|\right. \\
&- \frac{K_e}{K_\tau}\left(\frac{\omega(t_k) + \omega(t_{k+1})}{2}\right)(J\alpha(t_k+) + \tau_{load}(t_k+)) \\
&\left. - \left(\frac{J\alpha(t_k+) + \tau_{load}(t_k+)}{K_\tau}\right)^2 R\right)(t_{k+1} - t_k).
\end{aligned} \tag{31}$$

Expanding the summation yields

$$\begin{aligned}
P_{cont} =& \frac{1}{T}\left[\left(\frac{2B}{K_\tau}|J\alpha(t_1+) + \tau_{load}(t_1+)|\right.\right. \\
&- \frac{K_e}{K_\tau}\left(\frac{\omega(t_1) + \omega(t_2)}{2}\right)(J\alpha(t_1+) + \tau_{load}(t_1+)) \\
&\left. - \left(\frac{J\alpha(t_1+) + \tau_{load}(t_1+)}{K_\tau}\right)^2 R\right)(t_2 - t_1) \qquad (32) \\
&+ \cdots \\
&\left(\frac{2B}{K_\tau}|J\alpha(t_8+) + \tau_{load}(t_8+)|\right. \\
&- \frac{K_e}{K_\tau}\left(\frac{\omega(t_8) + \omega(t_9)}{2}\right)(J\alpha(t_8+) + \tau_{load}(t_8+)) \\
&\left.\left. - \left(\frac{J\alpha(t_8+) + \tau_{load}(t_8+)}{K_\tau}\right)^2 R\right)(t_9 - t_8)\right] \\
=& \frac{1}{1.8}\left[\frac{2 \cdot 30.4}{0.362} \cdot |0.0088 \cdot 523.6 + 0|\right. \\
&- \frac{0.362}{0.362}\left(\frac{0 + 104.7}{2}\right)(0.0088 \cdot 523.6 + 0) \\
&- \left(\frac{0.0088 \cdot 523.6 + 0}{0.362}\right)^2 \cdot 1.0)(0.2 - 0) \\
&+ \cdots \\
&+ \left(\frac{2 \cdot 30.4}{0.362}\right) \cdot |0.0088 \cdot 0 + (-1.5)| \\
&- \frac{0.362}{0.362}\left(\frac{0 + 0}{2}\right)(0.0088 \cdot 0 + (-1.5)) \\
&\left. - \left(\frac{0.0088 \cdot 0 + (-1.5)}{0.362}\right)^2 \cdot 1.0)(1.8 - 1.4)\right] \\
=& 384.6\, W.
\end{aligned} \tag{33}$$

This calculation does not apply to PWM amplifiers.

VII. Power Supply Sizing & Motor Ohmic Heating

The primary objective of this application note is to provide requirements useful for choosing an amplifier. It is possible, in addition, to compute power supply requirements and motor heat dissipation requirements since these additional requirements can be determined from the same design inputs used in the amplifier sizing equations.

A. Power Supply Sizing

Power supply sizing is accomplished with buses sized for peak power and current. For a linear amplifier, one has

$$P_{buslinear} = BI_{peak}. \tag{34}$$

The current per bus is

$$I_{buslinear} = I_{peak}. \tag{35}$$

For PWM amplifiers,

$$P_{busPWM} = 2BI_{peak}. \tag{36}$$

The current per bus is

$$I_{busPWM} = I_{peak}. \tag{37}$$

B. Motor Ohmic Heating $P_{motor-heat}$

The power input to the motor is converted to either mechanical power in the motor shaft or heat that must be transferred to the ambient environment. The dominant source of heat generated in the motor is the I^2R loss in the coil which is given by

$$P_{motor-heat} = I_{cont}^2 R. \tag{38}$$

There are other sources of heat generation in a motor associated with eddy-currents, friction, and hysteresis losses in motor iron. While these are generally negligible in a conservative thermal design, they can be included by adding power losses associated with viscous and Coulomb friction losses listed on the data sheet. Eddy-currents contribute to viscous losses, and hysteresis is usually lumped with friction measurements.

VIII. Conclusions

The equations of Section IV provide a simple means to size linear and PWM amplifiers for driving BTMs. In many applications, S-curves rather than lines define accelerations. The *peak output power* and *continuous power dissipation* calculations only apply to the sizing of linear amplifiers and such calculations are marked with an asterisk (*) throughout. Specifications on current are sufficient to estimate power dissipation in PWM amplifiers. The rounding of the corners by using S-curves generally reduces the demand on linear amplifiers. If one is attempting a close fit of the motion requirement to the amplifier specification, a more complete simulation (e.g. in Matlab) will provide valuable information. In the case of S-curves, the equations from Section III can be used. In such simulations more detailed modeling of inductance is also possible. Care should be taken with long-period motions and aperiodic motions as an assumption was made that the thermal time-constant of the heat sink systems is long enough that the heat sink temperature constant and is determined by P_{cont}.

Questions and feedback on this application note are most welcome and can be directed to sales@varedan.com.

References

[1] "Spreadsheet: Sizing Linear and PWM Amplifiers Driving a Brush-Type Motor," Varedan Technologies Document 4083-42-004.

[2] Hurley Gill, "Servomotor Parameters and their Proper Conversions for Servo Drive Utilization and Comparison," Kollmorgen Inc.